Title: The Ins and Outs of Polygons

Brief Overview:

The student will find the sum of both interior and exterior angles of polygons, the measure of each interior and exterior angle of regular polygons, and will be introduced to tessellations.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Mathematics instructional programs should focus on learning to reason and construct proofs as part of understanding mathematics so that all students make and investigate mathematical conjectures.

  Mathematics instructional programs also should use communication to foster understanding of mathematics so that all students organize and consolidate their mathematical thinking to communicate with others; express mathematical ideas coherently and clearly to peers, teachers, and others; extend their mathematical knowledge by considering the thinking and strategies of others; and use the language of mathematics as a precise means of mathematical expression.

- **Geometry and Spatial Sense**
  Mathematics instructional programs should include attention to geometry and spatial sense so that all students analyze characteristics and properties of two- and three-dimensional geometric objects; and recognize the usefulness of transformations and symmetry in analyzing mathematical situations.

Links to Virginia High School Mathematics Core Learning Units:

- **G.1**
  The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include translating a short verbal argument into symbolic form and using valid forms of deductive reasoning, including the law of syllogism.

- **G.2**
  The student will use pictorial representations, including computer software and coordinate methods to solve problems involving symmetry and transformation. This will include determining whether a figure has been translated, reflected, or rotated.

- **G.9**
  The student will use measures of interior and exterior angles of polygons to solve problems. Tessellation and tilting problems will be used to make connections to art, construction, and nature.

Grade/Level:

This lesson plan is designed for a high school Geometry course, Grades 8 - 12.

Duration/Length:

The lesson should cover 2-3 days of a 90-minute block or 4 days of a 50-minute class period.

Prerequisite Knowledge:

Students should have working knowledge of the Internet.

Objectives:

Students will:

- make conjectures about the exterior angles of a polygon.
- make conjectures about the interior angles of a polygon.
- identify transformations singly and in a tessellated image.

Materials/Resources/Printed Materials:

- Computer with internet access that is either Netscape 4.0 or later, or Explorer 4.0 or later on Windows
- Worksheets

Development/Procedures:

The students will access the Internet sites provided. The first worksheet will have them complete a vocabulary list of geometric terms using the Internet. The second worksheet will have them devise the conjecture concerning exterior angles of a polygon. The third worksheet will help them to devise the conjecture concerning the interior angles of the polygons. The fourth worksheet will introduce tessellation, having them analyze given tessellations and design their own.

Assessment:

This project is an introductory unit on polygons. The students will successfully complete the worksheets. Teachers may want to consider using problem # 19 on Worksheet #4 as an alternative form of assessment.

Extension/Follow Up:

- The students can design their own tessellation using the Geometer's Sketchpad or the TI-92.
- The students can investigate polygonal numbers.
- The students can take pictures of real life tessellations.
- The students can print out examples of tessellations found on the Internet.

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Teacher's Guide

Technology

- The ideal situation would be to have a computer lab available for your class with Internet access that is either Netscape 4.0 or later, or Explorer 4.0 or later in order to use the Java applet.
- An applet is an interactive program found on the Internet.
- We strongly recommend that the teacher **preview** the applet before using it in class.
- Allow ample time to load any sites that contain Java applets on your machine. While the applets are loading, you may want to do a 5-10 minute warm-up activity.
- Set a time limit for the students to do Worksheet #1. Do not let them surf the net too long.
- When using the applet in Worksheet #2, pressing the "Scale down" button continuously will reduce the polygon to a size such that the exterior angles will form a circle.
- Teachers may want to use Problem #19 on Worksheet #4 as an alternative assessment.
- If there is only one computer in your classroom, the teacher can lead the demonstration on a projected form of the monitor, if available, with student assisting throughout the activities.
- For your reference the following are used in this project:

  http://library.advanced.org/16284/index_s.htm
  http://tqd.advanced.org/2647/geometry/glossary.htm
  http://www.ies.co.jp/math/java/gaikaku/gaikaku.html
  http://forum.swarthmore.edu/sum95/suzanne/tess.intro.html
  http://www.geocities.com/SoHo/Museum/3828/tessellations.html
  http://www.best.com/~ejad/java/patterns/patterns_j.shtml

Worksheet #1 Name _____
Vocabulary

The following is a list of "must-know" vocabulary for this activity. You will use the Internet to complete the definitions and include the address of the site where you found the information. Please write in complete sentences. You may look anywhere you want for these definitions, but here are some suggested sites if you really get lost: http://library.advanced.org/16284/index_s.htm
http://tqd.advanced.org/2647/geometry/glossary.htm

diagonal:

exterior angle of a polygon:

heptagon:

hexagon:

interior angle of a polygon:

octagon:

pentagon:

polygon:

quadrilateral:

regular polygon:

triangle:

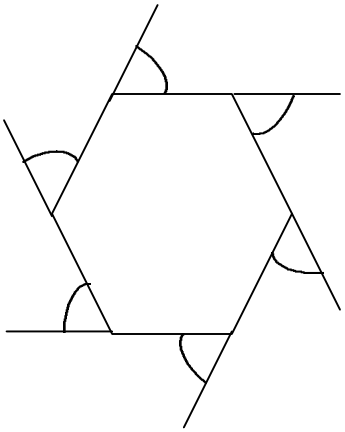
Worksheet #2 **Name** _____

Exterior Angles of a Polygon

1. Go to http://www.ies.co.jp/math/java/gaikaku/gaikaku.html

2. You should see **The Sum of Outer Angles of a Polygon**

3. Follow the steps given and complete the table.



About the applet

What is the sum of the angles?

Applet

a. Click some points to make a polygon.
b. Press "OK" button to close the polygon.
c. Press "Scale Down" button.
d. What do you find? (to start over, click on "Init"button)

For the triangle? _____

For the quadrilateral? _____

For the pentagon? _____

For the hexagon? _____

For the heptagon? _____

For the octagon? _____

4. Sketch each of the polygons that you made.

3 sides	4 sides	5 sides	6 sides
7 sides	8 sides		

5. Table

# sides	Sum of exterior angles	Measure of each exterior angle of regular polygon
3		
4		
5		
6		
7		
8		
. . .		
n		

6. Now generalize. Write a conjecture about the sum of the exterior angles of a polygon?

_____

Write a formula for a polygon with n-sides. _____

Now suppose that you are working only with regular polygons.

7. Complete the last column for the above table for the exterior angles. If the polygon is regular, what would be the measure of each exterior angle? _____

_____

Write a formula to find the measure of each exterior angle of a regular polygon with n-sides. _____

Worksheet # 3 **Name**_____

Interior Angles of a Polygon

1. Using the polygons that you drew for table in Worksheet # 2 on exterior angles, divide the polygons into triangles by drawing the diagonal(s) from one vertex and then complete columns 1-3 of the table.

# sides	# of diagonals drawn from 1 vertex	# triangles formed	Sum of interior angles	Measure of each interior angle of a regular polygon
3				
4				
5				
6				
7				
8				
. . .				
n				

2. What did you notice about the sum of the interior angles? _____

_____

3. Write a conjecture about the sum of the interior angles of a polygon. _____

_____

4. Write a formula for the sum of the interior angles of a polygon with n-sides. _____

Now suppose that you are working only with regular polygons.

5. Complete the last column for the above table. How did you find the answers? _____

_____

6. Write a conjecture dealing with the interior angle of a regular polygon. _____

_____

7. Write a formula to find the measure of each interior angle of a regular polygon.

_____

8. What is the most number of sides that could be used to make a regular polygon?

_____

9. What would be the measure, in degrees, of each interior angle (give a value) of the polygon in #8? _____

10. Finish this statement: *The more sides a polygon has, the more it would resemble a*

_____

Worksheet #4 Name _____

Tessellation

1. Go to **site A**: http://forum.swarthmore.edu/sum95/suzanne/tess.intro.html

2. Click on *"What Is a Tessellation?"*

3. Now tell me in your own words, what is a tessellation?

   tessellation:

4. What are the only three shapes that can tessellate an infinitely large plane?

5. Draw an example of part of a regular tessellation.

6. What *must* be true about an interior angle in a regular tessellation?
   Explain why this must be true.

7. Return to **site A**.

8. Click on *"Where's the Math?"*

9. Click on *"The Four Types of Symmetry in the Plane."*

10. In your own words, write a good definition of the following transformation terms:

    a. rotation:

b. translation:

c. reflection:

d. glide reflection:

11. Return to **site A**.

12. Click on *"Historical and Geographical Connections."*

13. Scroll down and click on *A Japanese Quilt Design*. This will show you some examples of more complex tessellations. Clicking on *"Next Java"* will allow you to see more examples. Look at a minimum of three different images. What is your favorite?

14. What basic shape is evident in each design?

15. Go to **site B**: http://www.geocities.com/SoHo/Museum/3828/tessellations.html

16. Pick one of the images by M. C. Escher. What sort of transformations are used to create the picture?

17. Using that same image, describe or sketch the single figure that is repeated in different colors or positions.

18. Go to **site C**: http://www.best.com/~ejad/java/patterns/patterns_j.shtml

19. Using rotations, translations, reflections, and glide reflections, fill the entire blank wall with your own wallpaper (tessellation) of triangles, rhombuses, trapezoids, and hexagons. Explain how you created your wallpaper using the four transformations. When you finish, be sure I see your final product.

    _____ teacher's initials

Answer Keys to Worksheets:

Worksheet #1

A diagonal is a segment that joins two non-consecutive vertices.

An exterior angle of a polygon is an angle that is adjacent and supplementary to one of the angles of a polygon.

A heptagon is a seven-sided polygon.

A hexagon is a six-sided polygon.

An interior angle of a polygon is an angle measured between two consecutive sides on the interior of a polygon.

An octagon is an eight-sided polygon.

A pentagon is a five-sided polygon.

A polygon is the union of three or more coplanar segments such that each endpoint is shared by exactly two segments. The segments intersect only at their endpoints and intersecting segments are noncollinear.

A quadrilateral is a four-sided polygon.

A regular polygon is a convex polygon that is both equilateral and equiangular.

A triangle is a three-sided polygon.

Worksheet #2

3d) For all polygons, the exterior angles create a circle and therefore the sum of the measures is 360°.

4) Answers vary.

5)Table

# of sides	Sum of exterior angles	Measure of each exterior angle of a regular polygon
3	360°	120°
4	360°	90°
5	360°	72°
6	360°	60°
7	360°	$51\frac{3}{7}^{0}$
8	360°	45°
n	360°	$\frac{360^{0}}{n}$

6) The sum of the measures of the exterior angles of a polygon is always 360°.
$S_e = 360°$

7) See last column of table for actual measures of regular polygons.
$M_e = \frac{360^{0}}{n}$

Worksheet #3

1) Table

# of sides	# of diagonals drawn from 1 vertex	# of triangles formed	Sum of interior angles	Measure of each interior angle of a regular polygon
3	0	1	180°	60°
4	1	2	360°	90°
5	2	3	540°	108°
6	3	4	720°	120°
7	4	5	900°	$128\frac{4}{7}^{\circ}$
8	5	6	1080°	135°
n	n-3	n-2	180°(n-2)	$\frac{180^{\circ}(n-2)}{n}$

2) The sum of the interior angles increases by 180° with each increase of the number of sides by one.

3) The sum of the interior angles of the polygon is 180° times the number of triangles formed by drawing all the diagonals from one vertex.

4) $S_i = 180°(n-2)$

5) See table for values.
To find each measurement, divide the sum of the interior angles by the number of sides of the polygon.

6) The interior angle of a regular polygon is the sum of all interior angles divided by the number of sides of the polygon.

7) $M_i = \frac{180^\circ(n-2)}{n}$

8) The most number of sides that could form a regular polygon is infinite.

9) The measure of the interior angle of the figure formed in #8 would approach 180°.

10) The more sides a polygon has, the more it would resemble a circle.

Worksheet #4

3) A tessellation is an arrangement of shapes in a pattern.

4) Only triangles, squares and hexagons can tessellate an infinite plane.

5) Examples should contain entirely triangles, entirely squares, or entirely hexagons.

6) The measure of an interior angle in a regular tessellation must be an exact divisor of 360. If it is not, there will be gaps or overlaps in the plane.

10a) A rotation involves turning an object around a center point at some given angle.

10b) A translation involves shifting an object in a certain direction at a given distance without turning or flipping it.

10c) A reflection involves flipping an object over a line.

10d) A glide reflection involves flipping an object over a line, then shifting that object along the line.

14) The figures in each image all take a basic hexagonal shape.

16) Answers vary, but should include rotation, translation, reflection, and glide reflection.

17) Answers are based on the image chosen.

19) Answers vary.